



# Nonlinear damping of a plate using Faraday instability of a fluid film

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## ABSTRACT

Damping using an instability of a fluid film in contact with a vibrating structure is investigated. Waves induced in the fluid film are the source of the added damping. A model based on the theory of Faraday instability is applied to a clamped circular plate covered by a fluid film. It is shown that this original technique can provide a significant damping, as with viscoelastic or porous material treatments. It is related to the amplitude of the waves which is a nonlinear function of the plate acceleration. Theoretical and experimental results are compared. The model overestimates the added damping: it is four times greater than the measured one.

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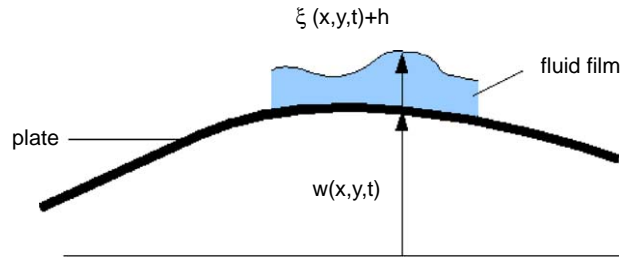
## 1. Introduction

This paper examines a method to reduce the vibration and therefore the emitted noise of a structure by means of a fluid film in the low frequency range. The usual techniques for noise and vibration reduction from a structure use a viscoelastic layer bonded onto the structure [1]. In this case, the dissipation is proportional to the loss factor of the material and to the flexural strain energy of the viscoelastic layer. To be efficient, this technique requires the use of a thick layer. Its thickness, and therefore the mass adding, can be reduced by using a light and stiff constraining sheet that increases the strain energy in the dissipating layer. Optimal partial covering may also be used to reduce the added mass [2]. These techniques are limited by viscoelastic properties that depend on frequency and temperature [3]. Designed primarily for sound absorbing, porous materials such as polymer foam may also add significant damping when mounted onto a structure [4–6]. To improve the efficiency of passive treatments, active control techniques have also been developed [7,8] but require more sophisticated set up. Moreover, their robustness has to be carefully demonstrated.

In this paper, damping added by a fluid film in contact with a structure is investigated (Fig. 1). When the normal acceleration of the structure is strong enough, stationary waves appear in the fluid film (Fig. 2). This phenomenon is called Faraday instability [9,10]. In case of a finite area of the fluid–air interface, the boundary conditions select countable wavelengths and several stationary mode shapes are solution of the problem [11]. The mode shape which has the greater amplification coefficient appears [12,13]. In case of an infinite area, this amplification coefficient will select the shape of the free surface among elementary cell patterns (roll, hexagon or square) (Fig. 2). These elementary cells can be considered as oscillators distributed over the surface of the structure [14], which damping depends on fluid flow in a cell and on the viscosity of the fluid. Moreover, the relation between the wave amplitude and the driving acceleration of the plate is nonlinear: it is necessary to determine the acceleration threshold for waves to appear and their amplitude at saturation [15,11]. Note that for high acceleration level, ejection of droplets can be observed [16]. This paper focuses on the added dissipation to the structure by the Faraday instability, using the thinnest fluid layer without droplet ejections. To the

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**Fig. 1.** Oscillation of a fluid film on a vibrating plate, with  $w(x,y,t)$  the transverse displacement of a point P of the plate (coordinates in the plane  $(x,y)$ ),  $t$  the time,  $h$  the water level of the fluid film at rest and  $\xi(x,y,t)$  the amplitude of the waves above the point P.



**Fig. 2.** Stationary waves at the fluid–air interface (top view of the plate). Driving acceleration  $W_d(0,0)$ :  $13.8 \text{ m s}^{-2}$ .

author’s knowledge, this technique aimed at reducing the vibration using the Faraday instability has not been previously presented.

In a first part (Section 2), the modelling of the method of calculating the added damping is detailed. To this end, a particular geometrical configuration is chosen: a circular plate clamped at its edge.

In Section 3, the corresponding experiment is designed in order to measure the added damping of the first mode of the plate. This highlights the nonlinear behaviour of the instability of the fluid film and the influence of the parameters governing the phenomenon.

The results of the model are compared with those obtained by the experiment in Section 4.

## 2. Model

### 2.1. From local to global dissipation

This section details how to calculate the global dissipation added on the structure as a function of the driving acceleration and for a given mode shape of the plate. Indeed, the modal dissipation is a function of the area where the instability appears and of the nonlinear relation between the amplitude of the waves and the local acceleration.

In the present paper, the first mode of a circular clamped plate of diameter  $d$  is considered. Its mode shape is given by [17]

$$\phi(r, \theta) = \frac{I_0(\beta_{01}d/2)J_0(\beta_{01}r) - J_0(\beta_{01}d/2)I_0(\beta_{01}r)}{I_0(\beta_{01}d/2)J_0(0) - J_0(\beta_{01}d/2)I_0(0)}, \quad (1)$$

with  $r$  the distance of the point to the centre of the plate,  $I_0$  the modified Bessel function,  $J_0$  the Bessel function and  $\beta_{01} = 1.0152\pi/d$ . This mode shape is normalized so that  $\phi(0, 0) = 1$ .

The modal damping ratio of the plate is given as a function of modal parameters,

$$\zeta_{ap} = \frac{c_{ap}}{2\sqrt{k_p m_{pf}}}, \quad (2)$$

where  $c_{ap}$  is the damping coefficient added on the plate,  $k_p$  the modal stiffness of the plate and  $m_{pf}$  the modal mass of the plate loaded by the fluid film. These three terms  $c_{ap}$ ,  $k_p$  and  $m_{pf}$  are calculated in the two following sections.

### 2.1.1. Modal stiffness and modal mass

The strain energy of the plate is calculated by integrating the local strain energy over the plate. This expression gives the modal stiffness  $k_p$  [17]:

$$\frac{1}{2}k_p(\phi(0, 0))^2 = \frac{1}{2}D \int_{\theta=0}^{2\pi} \int_{r=0}^R \left( \left( \phi_{,rr} + \frac{1}{r}\phi_{,r} \right)^2 - 2(1 - \nu_p) \frac{\phi_{,rr}\phi_{,r}}{r} \right) r dr d\theta, \quad (3)$$

with  $D$  the stiffness of the plate defined by

$$D = Ee^3/(2(1 - \nu_p^2)), \quad (4)$$

with  $E$  the Young modulus and  $\nu_p$  the Poisson's ratio of the material,  $e$  the thickness of the plate. This modal stiffness does not depend on the fluid film properties.

By the same approach, the modal mass of the plate  $m_p$  is given by [17]

$$\frac{1}{2}m_p(\phi(0, 0))^2 = \frac{1}{2} \int_{\theta=0}^{2\pi} \int_{r=0}^R \rho_p e \phi(r, \theta)^2 r dr d\theta, \quad (5)$$

with

$$\rho_p e = \rho_s e + \rho_f h = \rho_s e(1 + \tilde{\rho}) \quad (6)$$

the equivalent mass per unit area of the system, based on  $\rho_s$  the density of the aluminum,  $\rho_f$  the density of the fluid and an added mass due to the fluid layer. In this equation, the kinetic energy of the fluid due to the flow relative to the plate is neglected. Assuming that  $\rho_p$  is constant along the plate and that  $\phi(0, 0) = 1$ , Eq. (5) can be rewritten as

$$m_p = \rho_p e \int_{\theta=0}^{2\pi} \int_{r=0}^R \phi(r, \theta)^2 r dr d\theta. \quad (7)$$

The value of  $\rho_s$  is given by fitting the first resonance frequency of the bare plate,

$$f_1 = \frac{1.015^2}{2\pi} \frac{\pi^2}{(d/2)^2} \sqrt{\frac{D}{\rho_s e}}. \quad (8)$$

### 2.1.2. Added damping coefficient

The added damping coefficient  $c_{ap}$  is determined considering the dissipated energy during one period of the plate vibration [9,12,18–20],

$$\int_{t=0}^{2\pi/\omega_e} \frac{1}{2} c_{ap} \dot{w}^2(0, 0, t) dt = \int_{t=0}^{2\pi/\omega_e} \left[ \frac{1}{2} \int_{\theta=0}^{2\pi} \int_{r=0}^R \hat{c} \left( \frac{\partial h \alpha(r, \theta) \cos(\omega_f t)}{\partial t} \right)^2 r dr d\theta \right] dt, \quad (9)$$

with  $w(0, 0, t) = W \sin(\omega_e t)$  the instantaneous transverse displacement of the centre of the plate,  $W$  the amplitude of the displacement at this point,  $\hat{c}$  the local damping coefficient per unit area,  $\alpha(r, \theta) = A(r, \theta)/h$  (Eq. (12)) the dimensionless amplitude of the waves,  $A(r, \theta) = \max_t(\zeta(r, \theta, t))$  the amplitude of the waves at saturation (Fig. 2),  $\zeta(r, \theta, t)$  the instantaneous position of the free surface,  $\omega_f$  the circular frequency of the free surface waves due to the Faraday instability,  $\omega_e$  the circular frequency of excitation and  $t$  the time.

Taking into account the axisymmetry of the first mode shape, that  $\omega_f = \omega_e/2$  and  $\dot{w}(0, 0, t) = W\omega_e \sin(\omega_e t)$ , the integration in time and in angle  $\theta$  gives

$$c_{ap} W^2 = \hat{c} h^2 \pi \int_{r=0}^R \alpha(r)^2 r dr. \quad (10)$$

The integration is made on the area where the instability appears. It is a disk of radius  $r_d$ , so that for  $r > r_d$ ,  $\alpha(r) = 0$ . Assuming that the amplitude of the waves does not depend on the acceleration of the other points of the plate (local hypothesis),  $r_d$  may be defined by

$$\ddot{w}(r_d, \theta) = \tilde{\epsilon}_c g, \quad (11)$$

with  $\tilde{\epsilon}_c$  the dimensionless acceleration threshold for the Faraday instability to appear and  $g$  the gravity.

For  $r < r_d$ , a linear relation between  $\alpha^2$  and  $\tilde{\epsilon}$  is accounted for as suggested in Ref. [11]: the amplitude of the waves at saturation, when  $\partial A/\partial t = 0$ , is an affine function of  $\tilde{\epsilon}$ ,

$$\alpha^2 = \tilde{\epsilon}A_{\omega_f} - B_{\omega_f}, \tag{12}$$

where  $A_{\omega_f}$  and  $B_{\omega_f}$  are two parameters which will be determined experimentally in this paper (Section 3 and Fig. 7). Note that  $\tilde{\epsilon}_c$  is  $\tilde{\epsilon}$  for  $\alpha = 0$  in Eq. (12):

$$\tilde{\epsilon}_c = \frac{B_{\omega_f}}{A_{\omega_f}}. \tag{13}$$

Eq. (10) then reads

$$c_{\text{ap}} = \frac{\pi \hat{c} h^2}{W^2} \left[ A_{\omega_f} g \int_{r=0}^{r_d} \tilde{\epsilon}(r) r \, dr - B_{\omega_f} \int_{r=0}^{r_d} r \, dr \right], \tag{14}$$

where the dimensionless acceleration  $\tilde{\epsilon}(r)$  can be related to  $\phi(r, \theta)$  the normalized mode shape of the plate (Eq. (1)) by

$$\tilde{\epsilon}(r) = \frac{W \omega_e^2}{g} \phi(r, \theta). \tag{15}$$

### 2.2. Equivalent damping coefficient of a fluid cell

The aim of this section is to calculate the modal damping coefficient  $\hat{c}$  of the fluid per unit area, which is required to calculate  $c_{\text{ap}}$  (Eq. (14)). It is given by

$$\hat{c}(r, \theta) = 2\zeta_0 m_f \omega_f, \tag{16}$$

with  $m_f$  the modal mass per unit area and  $\zeta_0$  the damping ratio of the fluid.  $\zeta_0$  can be related to the logarithmic decrement  $\alpha_s$  of free oscillations [17] by

$$\zeta_0 = \frac{\alpha_s}{\sqrt{4\pi^2 + \alpha_s^2}}, \tag{17}$$

with  $\alpha_s = 2\pi\delta k$  [15]. Here  $\delta = \sqrt{2\nu/\omega_f}$  is the thickness of the viscous layer for a fluid with a kinematic viscosity  $\nu$  solicited at the circular frequency  $\omega_f$ , and  $k$  the wavenumber. This wavenumber is solution of [21]

$$\omega_f^2 = gk \tanh(kh) \left( 1 + \frac{\sigma k^2}{\rho_f g} \right), \tag{18}$$

with  $\sigma$  the surface tension of the fluid–air interface.

The modal mass per unit area  $m_f$  is

$$m_f = \frac{m_{f\text{cell}}}{(\lambda/2)^2}, \tag{19}$$

with  $m_{f\text{cell}}$  the modal mass of one cell whose area is  $(\lambda/2)^2$ , with  $\lambda = 2\pi/k$  the wavelength. Here, the first mode of a fluid cell for a given pattern of the free surface is considered. Several patterns appear successively on the free surface with the increase of the level of the acceleration [10]: roll, hexagon, then square. In this paper, the square pattern which is present for high accelerations is chosen (Fig. 2). The modal shape can be defined according to a potential function for the flow: the Reynolds is large enough ( $Re = 1.7 \times 10^6$ ) for the thickness of viscous boundary layer ( $\delta = 0.063$  mm) to be smaller than the fluid depth. Let us consider a fluid cell defined by a volume of incompressible fluid whose dimensions are  $\lambda/2$  in  $\mathbf{x}$  and  $\mathbf{y}$  directions, and  $h$  in  $\mathbf{z}$  direction. Symmetry arguments allow the use of a flow  $\mathbf{v}$  relative to the plate associated to  $\Phi_f(x, y, z, t) = \phi_f(x, y, z)g(t)$ , with  $g(t)$  a harmonic function and

$$\phi_f(x, y, z) = \beta(r, \theta) \cos(q(2x/\lambda - 1)) \cos(q(2y/\lambda - 1)) \cosh(\sqrt{2}q, z/\lambda), \tag{20}$$

with  $\beta$  the amplitude of the potential function which depends on the position of the fluid cell (cylindrical coordinates  $r, \theta$ ), and  $(x, y, z)$  the coordinates of a point into the cell. The velocity field  $\mathbf{v}$  is given by

$$h \frac{\partial \phi_f}{\partial x} g(t) = \mathbf{v} \cdot \mathbf{x}, \quad h \frac{\partial \phi_f}{\partial y} g(t) = \mathbf{v} \cdot \mathbf{y}, \quad h \frac{\partial \phi_f}{\partial z} g(t) = \mathbf{v} \cdot \mathbf{z}. \tag{21}$$

The boundary conditions at  $x = \lambda/2$  and  $y = \lambda/2$  give for the first mode of the fluid cell,  $q = \pi$ .

The kinetic energy of the fluid in the cell is used to calculate  $m_{f\text{cell}}$ :

$$\frac{1}{2} m_{f\text{cell}} \left( \frac{\partial \alpha(r, \theta) \cos(\omega_f t)}{\partial t} \right)^2 = \frac{1}{2} \rho_f g(t)^2 \int_{x=0}^{\lambda} \int_{y=0}^{\lambda} \int_{z=0}^h (\phi_{f,x}^2 + \phi_{f,y}^2 + \phi_{f,z}^2) \, dx \, dy \, dz, \tag{22}$$

with  $\phi_{f,x} = \partial f / \partial x$ .  $\beta(r, \theta)$  is derived from  $\alpha(r, \theta)$  the amplitude of the waves, by equating the vertical velocity of a point of the free surface ( $z = h$ ):

$$\alpha(r, \theta) \frac{\partial \cos(\omega_f t)}{\partial t} = g(t) \frac{\partial \phi_f}{\partial z} \quad \forall t, \quad (23)$$

and using Eq. (20) it gives

$$\alpha(r, \theta) = \beta(r, \theta) \frac{2\sqrt{2}\pi}{\lambda} \sinh(\sqrt{2}\pi h / \lambda), \quad (24)$$

$$g(t) = -\omega_f \sin(\omega_f t) \quad (25)$$

so that

$$\beta(r, \theta) = \frac{\alpha(r, \theta)\lambda}{2\sqrt{2}\pi \sinh(\sqrt{2}\pi h / \lambda)}. \quad (26)$$

Using Eq. (26), and integrating the second member of Eq. (22), the modal mass of the one degree of freedom system associated to the fluid cell writes

$$m_{f\text{cell}} = \frac{\rho_f \lambda^3 \sqrt{2}}{256\pi} \frac{\left(1 - \exp\left(\frac{-8\sqrt{2}\pi h}{\lambda}\right)\right) \exp\left(\frac{4\sqrt{2}\pi h}{\lambda}\right)}{\cosh\left(\frac{\sqrt{2}\pi h}{\lambda}\right)^2 - 1}. \quad (27)$$

The modal mass per unit area  $m_f$  and the modal damping coefficient by unit area  $\hat{c}$  may be obtained with Eqs. (16) and (19).

### 3. Experimental results

The aim of this section is to determine experimentally the plate damping induced by the Faraday instability of the fluid film. Because the system is nonlinear, all experimental quantities are referenced to the acceleration at the centre of the plate, called driving acceleration, and denoted  $W_a(0, 0)$ . Four water levels and four values of the driving acceleration are considered.

#### 3.1. Experimental set-up

The geometrical configuration is presented in Fig. 3. A circular aluminum plate of diameter  $d = 0.290$  m is clamped. Its characteristics are summarized in Tables 1 and 2. It is excited by a shaker connected to its centre via a force transducer giving the excitation force  $F$ . An accelerometer of mass  $m_a = 0.0042$  kg is bonded at 1 cm of the centre of the plate to get the reference acceleration  $W_a(0, 0) = \omega_c^2 W$ . A laser vibrometer is focused on the plate through the fluid film. The laser spot area is of the order of  $1 \text{ mm}^2$ .

The frequency range is set between 40 and 120 Hz to ensure Faraday instabilities for experimentally achievable accelerations. The thickness of the fluid layer is chosen to be of the order of the wavelength: this ensures a total covering of the plate and a small added mass.

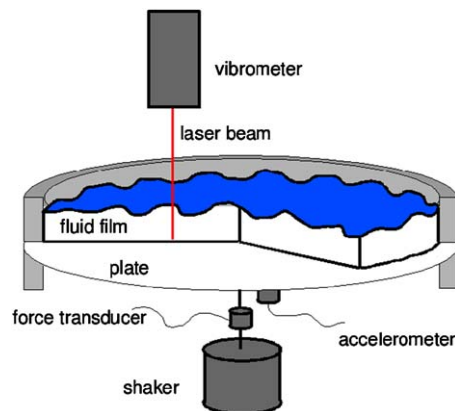


Fig. 3. Experimental set-up.

**Table 1**  
Numerical inputs of the model.

<i>Plate</i>		
Diameter of the plate (m)	$d$	0.290
Thickness of the plate (m)	$e$	0.001
Material of the plate		Aluminum
Young's Modulus (Pa)	$E$	$7 \times 10^{10}$
Poisson's ratio	$\nu_p$	0.3
Frequency of the first resonance of the bare plate (Hz)	$f_1$	100
Frequency of excitation (Hz)	$f_e$	75
<i>Fluid</i>		
Gravity ( $\text{m s}^{-2}$ )	$g$	9.81
Fluid layer thickness (m)	$h$	0.0046
Kinematic viscosity of water ( $\text{m}^2 \text{s}^{-1}$ )	$\nu$	$10^{-6}$
Density of the fluid ( $\text{kg m}^{-3}$ )	$\rho_f$	1000
Wavelength of the free surface (m)	$\lambda$	0.0063
First amplitude of the wave coefficient	$A_{\omega_f}$	[0.0135,0.0163]
Second amplitude of the wave coefficient	$B_{\omega_f}$	[0.0111,0.0134]

**Table 2**  
Numerical outputs of the model.

<i>Loaded plate</i>			
Stiffness of the plate (N m)	Eq. (4)	$D$	8.82
Dimensionless added mass	Eq. (30)	$\tilde{\rho}$	0.88
Density of the plate with air loading ( $\text{kg m}^{-3}$ )	Eq. (8)	$\rho_s$	5230
Modal stiffness ( $\text{Nm}^{-1}$ )	Eq. (3)	$k_p$	1.08105
Equivalent mass of the plate with fluid ( $\text{kg m}^{-3}$ )	Eq. (6)	$\rho_p$	7215
Modal mass (kg)	Eq. (7)	$m_p$	0.0875
<i>Fluid</i>			
Threshold of instability ( $\text{m s}^{-2}$ )	Eq. (13)	$\tilde{e}_c g$	8.04
Local damping ratio of the fluid	Eq. (17)	$\zeta_0$	14.5%
Modal mass of a fluid cell (kg)	Eq. (27)	$m_{f\text{cell}}$	$2.23 \times 10^{-6}$
Modal mass per unit area ( $\text{kg m}^{-2}$ )	Eq. (19)	$m_f$	$2.25 \times 10^{-1}$
Equivalent damping coefficient per unit area ( $\text{kg m}^{-2} \text{s}^{-1}$ )	Eq. (16)	$\hat{c}$	9.69
Damping ratio added by the fluid	Eq. (2)	$\zeta_{\text{ap}}$	Fig. 6

The frequency response functions are obtained using a step by step harmonic excitation for a constant acceleration amplitude  $W_a(0,0)$ .

### 3.2. Determination of the free surface velocity of the fluid

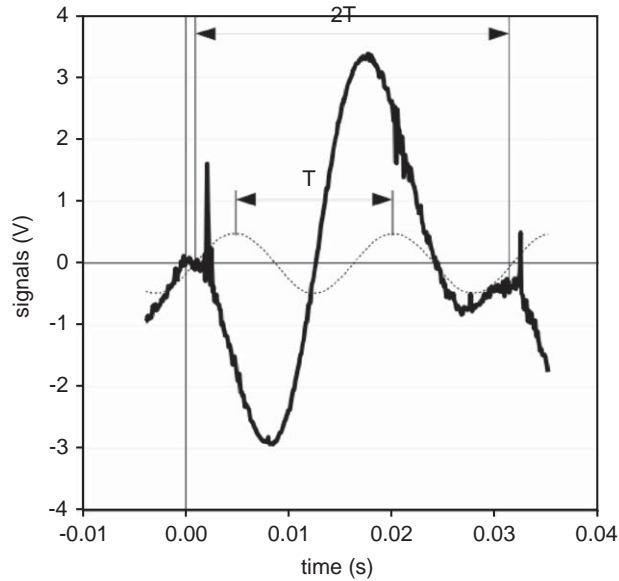
The signal given by the laser vibrometer is proportional to the apparent velocity of the plate  $d\tilde{w}/dt$ . It is a combination of the plate and of the free surface velocities:

$$\frac{d\tilde{w}(r, \theta, t)}{dt} = \frac{dw(r, \theta, t)}{dt} - \left(\frac{c_0}{c_1} - 1\right) \frac{d(h + \zeta(r, \theta, t))}{dt}, \tag{28}$$

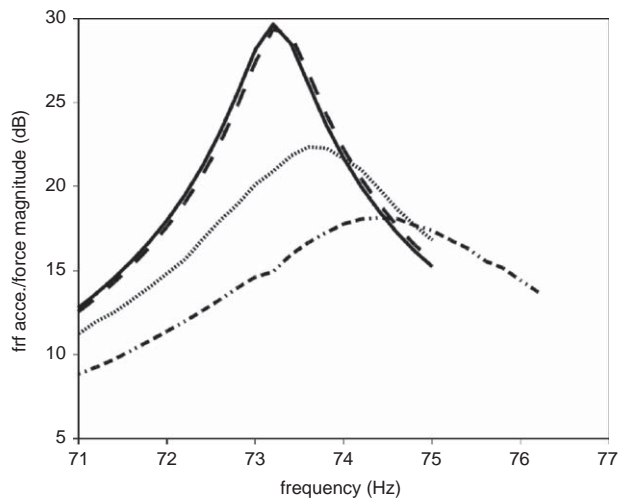
with  $w(r, \theta, t)$  the displacement of the plate at the point  $P(r, \theta)$ ,  $c_0$  the light velocity in the air,  $c_1$  the light velocity in the fluid,  $h(r, \theta, t)$  the water level crossed by the beam. The time signal of the apparent speed of the plate (Fig. 4) exhibits a sub-harmonic at  $\omega_e/2$ . Their contributions can be easily identified because they are at different frequencies (see Fig. 4). To do so, Eq. (28) is rewritten as

$$\frac{d\tilde{w}(r, \theta, t)}{dt} = \tilde{a}_1 \cos(\omega_e t + \tilde{a}_2) + \tilde{a}_3 \cos\left(\frac{\omega_e}{2} t + \tilde{a}_4\right). \tag{29}$$

The parameters  $\tilde{a}_1$  and  $\tilde{a}_2$  are associated to the plate velocity, and the parameters  $\tilde{a}_3$  and  $\tilde{a}_4$  to the fluid interface movement. They are determined using a nonlinear optimization. The position of the nodes of the free surface mode shape is not stationary:  $\tilde{a}_3 = 0$  if the laser beam focuses through a node, and is maximum if the laser beam focuses through an



**Fig. 4.** Signals (V) of the acceleration of the plate (dotted line  $31.6 \text{ m s}^{-2}/\text{V}$ ) and its apparent velocity measured by the laser vibrometer (bold line  $0.025 \text{ m s}^{-1}/\text{V}$ ) maximum velocity of the plate:  $0.039 \text{ m s}^{-1}$ ; water level  $0.0046 \text{ m}$ ;  $\tilde{\rho} = 0.88$ ; frequency of excitation  $65 \text{ Hz}$ .



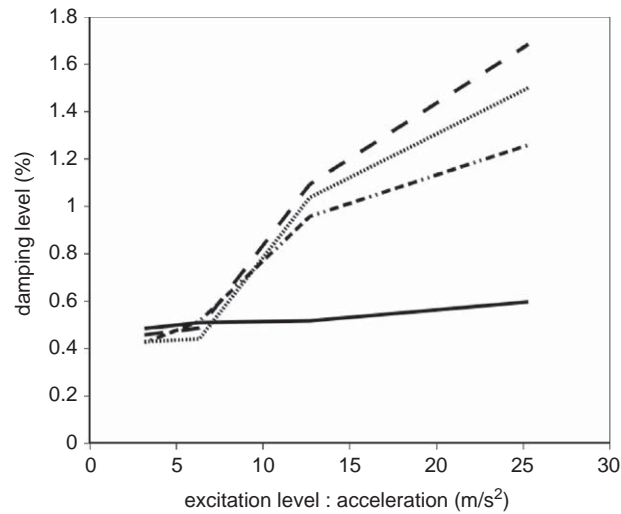
**Fig. 5.** Influence of the driving acceleration  $W_a(0,0)$  on the frequency response function (accelerometer/force) for a water level of  $0.0046 \text{ m}$ ,  $\tilde{\rho} = 0.88$ : continuous line,  $W_a(0,0) = 3.16 \text{ m s}^{-2}$ ; dashed line,  $W_a(0,0) = 6.32 \text{ m s}^{-2}$ ; dotted line,  $W_a(0,0) = 12.7 \text{ m s}^{-2}$ ; dashed dotted line,  $W_a(0,0) = 25.3 \text{ m s}^{-2}$ .

antinode. Indeed, for level of accelerations far higher than the threshold, second-order effects [22] induce a slow drift of the positions of the antinode of the fluid–air interface. Thus, the time of acquisition must be long enough to contain measurement on an antinode. The amplitude of the waves is then deduced from the greatest value of  $\tilde{a}_3$ .

### 3.3. Influence of the driving acceleration and water levels

The experiments are performed with water to test the influence of the fluid level on the frequency response functions. The water levels remain below  $0.0133 \text{ m}$  and above  $0.004 \text{ m}$  for all the experiments. Below this lower limit, the wetting mechanism prevents the water from covering the whole plate. The ratio of added mass per unit area is

$$\tilde{\rho} = \frac{\rho_f h}{\rho_s e}. \quad (30)$$



**Fig. 6.** Influence of the driving amplitude  $W_a(0, 0)$  on the plate damping for several water levels: continuous line, 0.0 m; dashed line, 4.6 mm; dotted line, 8.0 mm; dashed dotted line, 13.3 mm.

Near the resonance frequency (75 Hz), stationary waves are observed on the free surface (Fig. 2). Their amplitude is higher at the centre of the plate. This coincides with the maximum amplitude of the first mode shape. They appear as square patterns within a circle at the centre of the plate. Their wavelength, which are consistent with capillarity waves on the surface, increases as the frequency decreases.

To determine the acceleration threshold for Faraday instability, the frequency response function is measured for several amplitudes of the driving acceleration for water levels from 0 to 0.0133 m. Nonlinear effects are observed (Fig. 5) when increasing the amplitude of acceleration:

- the existence of a threshold for waves to appear,
- an increase in the area of the circular surface on which waves are present,
- a slight increase in the frequency resonance,
- a reduction of the amplitude of the peak (up to 13 dB).

In Fig. 5, the threshold is between 6 and 13 m s<sup>-2</sup>. It corresponds to a sharp increase in damping. Fig. 6 shows damping ratio  $\zeta_{pf}$  which is estimated using the half-power method. The measured damping for this non-optimized configuration can be half the value of the damping measured in the case of unconstrained-layer treatment, and is greater than the damping added by foam or fibre layer. This increase is not observed for the bare plate (continuous line). The thinner the fluid layer, the stronger the damping is.

#### 4. Comparison between theoretical and experimental results

Comparison between theoretical and experimental values of the plate damping induced by the fluid film is now evaluated. Water with the thinner thickness is used to induce the stronger damping. The numerical values of the parameters of the model are given in Tables 1 and 2 for this configuration.

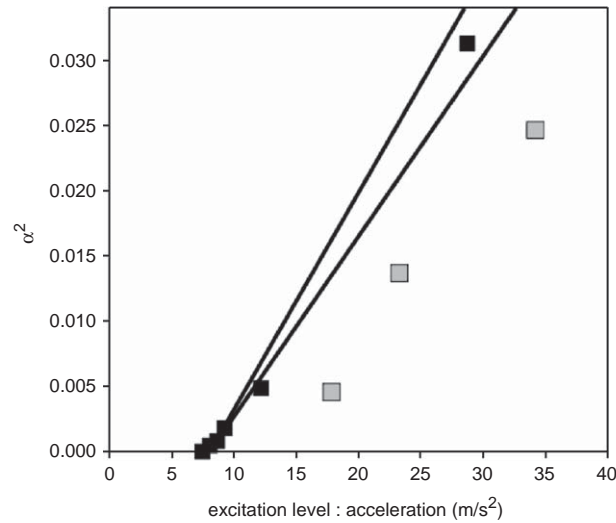
The added damping ratio depends on the local dissipation of the fluid layer and on the area of the instability area.

This local dissipation is a function of the relation between the local acceleration of the plate and of the wave amplitude (Fig. 6). The relation between the wave amplitude and the motion of the plate is quantified by the coefficients  $A_{\omega_f}$  and  $B_{\omega_f}$  (Eq. (12)) which are identified on the experimental data by a least-squares regression method (Fig. 7). The uncertainties on the position of the regression line are calculated with a confidence of 95%. Note that three points are not taken into account: the noise on the signal and the low spatial stability of the waves do not allow a correct evaluation of the amplitude of the waves for these three acceleration levels. In the following, above the threshold  $\tilde{\epsilon}_c$ , the dimensionless amplitudes of the waves lie in the following boundaries:

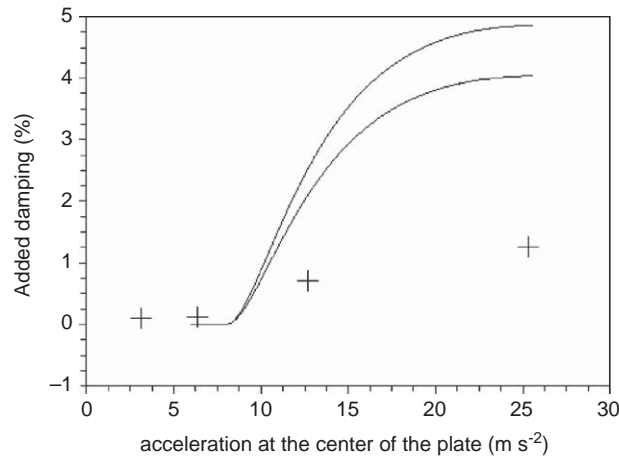
$$0.0135(\tilde{\epsilon} - 0.82) < \alpha^2 < 0.0163(\tilde{\epsilon} - 0.82). \quad (31)$$

The wavelength  $\lambda = 0.004$  m given by the model using  $\sigma = 0.072$  N m<sup>-1</sup>, the surface tension of the fluid–air interface, is near the measured experimental wavelength ( $0.00626$  m  $< \lambda < 0.00634$  m, Section 3.3). The experimental average value  $\lambda = 0.0063$  m is used in the model to evaluate the damping.





**Fig. 7.** Experimental evolution of the dimensionless oscillation amplitude  $\alpha = A/h$  with the acceleration above the threshold for a given frequency (75 Hz) with water. The two lines correspond to the uncertainty with a confidence of 95% of the mean square line which fits the data.



**Fig. 8.** Evolution of the damping ratio added by the fluid versus the driving acceleration  $W_a(0,0)$ : theoretical values (between the two lines: the uncertainties on the measurement of the water waves induce two curves of dimensionless damping with 95% of confidence), experimental values (crosses).

Moreover, the equivalent damping coefficient  $c_{ap}$  added to the structure depends on the area on which the damping occurs. The first mode shape of the plate induces a circular limit of instability of radius  $r_d$  where  $W_a(r_d, \theta) = 8.04 \text{ m s}^{-2}$ . The relationship between this radius and the acceleration in the centre of the plate is given by the mode shape of the plate  $\phi(r, \theta)$  (Eq. (1)) and the instability threshold. The evolution of  $c_{ap}$ , as function of  $W_a$  at the centre of the plate, can then be calculated taking into account the uncertainty of the amplitude of the waves (Eq. (31)). From the evolution of  $c_{ap}$ , the damping ratio added by the fluid  $\zeta_{ap}$  can be plotted (Fig. 8).

For the experimental point of view, the damping ratio added by the fluid can be deduced by the comparison of the damping ratio observed with the fluid  $\zeta_{pf}$  and the damping ratio of the bare plate  $\zeta_p$  (Fig. 6). The damping ratio of the bare plate is,

$$\zeta_p = \frac{c_p}{2\sqrt{k_p m_p}}, \quad (32)$$

with  $c_p$  the modal damping coefficient of the plate,  $m_p$  the modal mass of the plate. With the fluid layer the damping ratio is given by

$$\zeta_{pf} = \frac{c_p + c_{ap}}{2\sqrt{k_p m_{pf}}} = \frac{c_p}{2\sqrt{k_p m_{pf}}} + \zeta_{ap}. \quad (33)$$

By neglecting the kinetic energy of the waves compared to the kinetic energy of the loaded plate, the relation  $m_{pf} = m_p(1 + \bar{\rho})$  gives

$$\zeta_{pf} = \frac{c_p}{2\sqrt{k_p m_p(1 + \bar{\rho})}} + \zeta_{ap}. \quad (34)$$

Thus, the experimental value of the added damping ratio is

$$\zeta_{ap} = \zeta_{pf} - \zeta_p \frac{1}{\sqrt{1 + \bar{\rho}}}. \quad (35)$$

The comparison (Fig. 8) between experimental and theoretical values of  $\zeta_{ap}$  shows that above the threshold the amplitude of the theoretical damping is four times greater than the experimental damping.

The source of this discrepancy can be sought in the high sensitivity of the model to the amplitudes of the waves. The relation between the wave amplitude and the acceleration has been determined assuming a locally reacting behaviour of each fluid cell. This hypothesis is valid where the acceleration is uniform: this is the case in the vicinity of an antinode (centre of the plate for the first mode). This assumption may not be valid where an acceleration gradient is present. Moreover, in this transition region, the cell shape is no longer square (Fig. 2).

Nevertheless, note that the damping added by the fluid film is comparable to those obtained with viscoelastic or porous material treatments [1,6].

## 5. Conclusion

Damping of a vibrating structure by means of a heavy fluid film subjected to Faraday instability has been studied. It is shown that added damping may be significant. A model has highlighted the governing parameters: the acceleration threshold and the amplitude of the waves, which depend on the frequency, on the viscosity and the fluid thickness of the fluid and on the mode shape of the structure. The model takes into account that the dissipation does not act on the overall structure. Experimental results have confirmed the tendencies of the predicted damping. Nevertheless, the model overestimates the added damping: it is four times greater than the measured one. Further experiments are needed to detect the sources of discrepancies: the Faraday instability threshold and the amplitude of the waves must be studied in the case of non-uniform acceleration.

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